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A combinatorial proof of Shapiro's Catalan convolution ☆,☆☆

Gábor V. Nagy

Bolyai Institute, University of Szeged, Szeged, H-6720, Hungary

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ABSTRACT

Shapiro proved an elegant convolution formula involving Catalan numbers of even index. This paper gives a combinatorial proof of his formula. In addition, we show that it is equivalent to an alternating convolution formula of central binomial coefficients.

1. Introduction

In this paper C_n denotes the nth Catalan number and B_n denotes the nth central binomial coefficient, i.e. $C_n = \frac{1}{n+1}\binom{2n}{n}$ and $B_n = \binom{2n}{n}$. Unless otherwise stated, all indices (i, j, k, and so on) are nonnegative integers in our formulas.

In 2002, L. Shapiro found the following elegant identity [3, p. 123]:

Theorem 1.

$$\sum_{i+j=n} C_{2i}C_{2j} = 4^n C_n. \tag{1}$$

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E-mail address: ngaba@math.u-szeged.hu.

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